Guarantees in Fair Division, under informational parsimony

Hervé Moulin

Abstract

Steinhaus' Diminishing Share (DS) algorithm (generalizing Divide & Choose D&C), as well as Dubins and Spanier' Moving Knife (MK) algorithm, guarantee to all participants a Fair Share of the manna (its worth at least 1/n-th of that of the whole manna) while eliciting parsimonious information from them. However DS and MK are only defined when 1. preferences are represented by additive utilities.; and 2. every part of the manna to be divided is desirable to every participant (a cake), or every part is unpleasant to everybody (a chore). Our *n-person Divide & Choose* rule takes care of issue 2 when utilities are additive: it requires no trimming or padding, and works for mixed manna with subjective goods and bads. It also implements the canonical approximation of the Fair Share (up to one item) when we allocate indivisible items. Issue 1 is much deeper, it challenges us to define a Fair Share Guarantee when 1/n-th of the whole manna makes no sense. The same D&C$_n$ rule implements such a bound, for very general preferences restricted by a continuity assumption but no monotonicity whatsoever. The *minMax utility* of an agent is that of his best share in the worst possible partition. It is lower than his Maxmin utility (that of his worst share in the best possible partition), that cannot be guaranteed to all agents. When the manna contains only goods, or only bads, the minMax Guarantee can be improved in infinitely many ways. Our *Bid & Choose* rules improve upon the MK rules by fixing a benchmark value of shares, and asking agents to bid the smallest size of an acceptable share. The resulting Guarantees fall between their minMax and Maxmin.