

# Balanced collections revisited

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## Abstract

Balanced collections of subsets of a finite set were introduced in the 1960's by Lloyd Shapley as a means to describe those cooperative  $n$ -person games having a nonempty core, that is, those games for which the coalition of the whole had enough resources to distribute so that no subset of the players could do better on its own. Minimal balanced collections provide an irredundant description of such games.

Collections that are not balanced are called “unbalanced”. In recent years, the question of enumerating maximal unbalanced collections has arisen in a variety of fields, for example in physics in “thermal field theory”. I will discuss some consequences of attempts to solve this difficult enumeration problem.

In 1972, Michel Balinski studied a more general property of collections than being balanced that he called “harmonious”; in particular, he showed balanced collections are harmonious. It is unknown whether the corresponding enumeration question for “unharmonious collections” might be easier to solve.